Fleet Behaviour, Management Plans and Endogenous Fishing Mortalities Fluctuations *

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Abstract

We evaluate the robustness of F bands to boilogical fluctuations due by stochastic recruitments. To this end we develop a multi-fleet state-space model where fishing mortality is the endogenous result of the fleets' responses to the management measures.

1 Introduction

We evaluate the robustness of F bands to boilogical fluctuations due by stochastic recruitments. To this end we develop a multi-fleet state-space model where fishing mortality is the endogenous result of the fleets' responses to the management measures. We show how fishing mortality interactions can be modelled as a simple costate dependent variables in a multi-fleet age-structured model. The model can be used to simulate the size of the fishing mortalities fluctuations around reference (equilibrium) targets.

New Generation of Management Plans establish that the fishing mortality rates can float within

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certain margins of fluctuations around a fixed central rate in line with the MSY principle. Once the lower and upper bound of the band are fixed, managers only intervene to avoid excessive mortality rates fluctuations. For example the limits for the North See Cod are given by $F \in (0.13 - 0.33) \equiv F_{max} \times (1 \pm .5)$

Our paper is close to those articles that split the fishing activity in different components (fleets, fishery management unit, métiers, segments, ..) in order to attend the complexity derived from mixed stock advice (Katsanevakis *et al.*, 2010; Da Rocha *et al.*, 2012; Da Rocha and Gutiérrez, 2012, Ulrich *et al.*, 2002; Simons et al., 2014).

The inclusion of fleets in the management decisions can be addressed using a multi-stage approach. In a multi-stage context the decisions taking by agents at a first stage are independent of other agents' behavior. And at stage two, agents use decision from stage one to decide on the best strategy. This kind of approach has been used for answering different issues in fisheries management (Ruseski, 1998; Kronbak and Lindroos, 2006; Da Rocha and Gutiérrez, 2012; van Dijk *et al.*, 2014). In this article we consider a stochastic multi-fleet model where agents take decisions in a multi-stage context. Roughly speaking, the behavior of each fleet endogenously determines the aggregated performance of the fishery. Different from Stouten *et al.* (2008) the model incorporates the biological component of the fish population. And unlike Homans and Wilen (1997) and Da Rocha and Gutiérrez (2012) our bioeconomic model considers an age-structured biological model.

The paper proceeds as follows. In the next section a multi-fleet age-structured model is presented is presented. Section 3 shows a numerical application for the European Southern Hake Stock. Finally, Section 5 concludes the paper.

2 A multi-fleet age-structured model

We consider a stochastic multi-fleet model where the behavior of each fleet endogenously determines the aggregated performance of the fishery. The model represents a bi-level decision problem. At a first level, the fishery regulator decides on an effort variable such as maximum fishing days taking as given the aggregated fishing mortality. At the second level, fleets reveal short-term behavior by reacting to the maximum fishing days determined by the regulator and taking as given the aggregate fishing mortalities by age. In equilibrium, the aggregated variables of the fishery are determined by the behavior of all the agents.

2.1 Aggregated population dynamics

Consider a fishery with A age-classes being $N_{a,t}$ the fish population of age a in period t, respectively. The dynamics of the population is given by

$$N_{a+1,t+1} = e^{-Z_{a,t}} N_{a,t}, \tag{1}$$

where subscripts $a = 1, 2, 3$ and t represents the age and time. Equation (1) considers that population of any age decreases at an exponential rate in accordance with the mortality rate $Z_{a,t}$. This mortality rate by age which is decomposed into fishing mortality, $F_{a,t}$, and natural mortality (non-human predation, disease and old age), m_a. So, we can express aggregate mortality by age as

$$Z_{a,t} = F_{a,t} + m_a. \tag{2}$$

Moreover a usual assumption in assessment fisheries models is that the fishing mortality over each age is given by stationary selectivity patterns, p_a, i.e.

$$F_t^a = p_a F_t. \tag{3}$$

Assume that each period, t, a stochastic exogenous number of juvenile are born. We normalize the expected number of recruits equal to 1. In particular, we assume that recruitment follows the next stochastic process

$$N_{1,t} = e^{x_{1,t}}$$

where $x_{1,t}$ is an AR(1) process

$$x_{1,t+1} = \rho x_{1,t} + \varepsilon_{t+1},$$

being ε a white noise with variance σ_ε. The parameter ρ is the correlation coefficient that defines the persistence relationship between the recruitments today versus tomorrow.

Let $x_{a,t}$ be the logarithm of the variable $N_{a,t}$ and $\Delta x_{a,t}$ and $\Delta Z_{a,t}$ be their difference with respect to the steady state values. Formally, $x_{a,t} = LnN_{a,t}$, $\Delta x_{a,t} = x_{a,t} - \overline{x_a}$ and $\Delta Z_{a,t} = Z_{a,t} - \overline{Z_a}$ where the bar over the variable represents the steady state value. With this notation, the dynamic model can be expressed in matrix form. In particular, for the case of three ages the following system fully represents the dynamics of the population,

$$\begin{bmatrix} \Delta x_{1,t+1} \\ \Delta x_{2,t+1} \\ ... \\ \Delta x_{A,t+1} \end{bmatrix} = \begin{bmatrix} \rho & 0 & ... & 0 & 0 \\ 1 & 0 & ... & 0 & 0 \\ 0 & 1 & ... & 0 & 0 \\ ... & ... & ... & ... & ... \\ 0 & 0 & ... & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta x_{1,t} \\ \Delta x_{2,t} \\ \\ \Delta x_{A,t} \end{bmatrix} - \begin{bmatrix} 0 & 0 & ... & 0 & 0 \\ 1 & 0 & ... & 0 & 0 \\ 0 & 1 & ... & 0 & 0 \\ ... & ... & ... & ... & ... \\ 0 & 0 & ... & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta Z_{1,t} \\ \Delta Z_{2,t} \\ ... \\ \Delta Z_{A,t} \end{bmatrix}.$$

In a more compact way the dynamic system can be represented, with the obvious definitions, as

$$\Delta \mathbf{x}_{t+1} = \mathbf{A}\Delta \mathbf{x}_t - \mathbf{B}\Delta \mathbf{Z}_t. \tag{4}$$

The total biomass of the fishery for a particular period is given by

$$B_t = \sum_{a=1}^{A} \mu_a N_{a,t} = \sum_{a=1}^{A} \mu_a e^{x_{a,t}},$$

where μ_a represents the weight of age a.

Aggregated yield by age is given by the Baranov equation (1918). That is

$$Y_{a,t} = \frac{p_a F_t}{Z_{a,t}} \left(1 - e^{-Z_{a,t}}\right) e^{x_{a,t}}. \tag{5}$$

Therefore, the total yield of the fishery is defined as $Y_t = \sum_{a=1}^{A} Y_{a,t}$.

2.2 Fleet's behavior

The fishery is formed by N fleets which have different technologies. Each fleet is composed by a continuum of identical vessels with measure one. Vessels decide on effort that maximizes their profits. Since the profit function we consider is homogenous of degree one, we can represent the decision of each fleet by analyzing the problem of a representative vessel that takes as given the aggregate variables of the fishery.

The fishery is open during a maximum of T days fixed by the regulator. The profits of the representative vessel of fleet f for any operating day t are given by the difference between revenues, $\sum_{a=1}^{A} \frac{p_{a,f} e_{f,t}}{Z_{a,t}} \left(1 - e^{-Z_{a,t}}\right) e^{x_{a,t}}$, and fishing cost $TC_{f,t} = \frac{1}{2} c_f e_{f,t}^2$. Profits are expressed in real terms. Captures of each fleet by age are calculated using the Baranov equation (1918). Notice that we are considering a convex cost function where the parameter c_f represents its curvature. The higher c_f is the higher the increase of the costs due to changes in effort is.

Formally the representative vessel of fleet f maximizes its profits taking as given the aggregates of the fishery. That is, each vessel f solves

$$\max_{e_{f,t}} \quad \pi_{f,t} = \sum_{a=1}^{A} \frac{p_{a,f} e_{f,t}}{Z_{a,t}} \left(1 - e^{-Z_{a,t}}\right) e^{x_{a,t}} - \frac{1}{2} c_f e_{f,t}^2.$$

The first order condition of this maximization problem is given by

$$c_f e_{f,t} = \sum_{a=1}^{A} \frac{p_{a,f}}{Z_{a,t}} \left(1 - e^{-Z_{a,t}}\right) e^{x_{a,t}}. \tag{6}$$

Condition (6) shows how the effort is selected by each fleet given the aggregate mortality rates, $Z_{a,t}$, and the population driven by the socks on recruitments, $x_{a,t}$. The optimal effort selected by each fleet is the one for which the marginal cost (left hand side) equals to the marginal revenue (right hand side).

Linearizing condition (6) around the steady state values we can express the optimality condition in matrix form as

$$\Delta e_{f,t} = C_{f,t}\Delta \mathbf{x}_t + D_{f,t}\Delta \mathbf{Z}_t, \tag{7}$$

where $\Delta e_{f,t} = e_{f,t} - \overline{e_f}$, and $C_{f,t} = \left[\begin{array}{ccc} \frac{\partial e_{f,t}}{\partial x_{1,t}}, & ...., & \frac{\partial e_{f,t}}{\partial x_{A,t}} \end{array} \right]$ and $D_{f,t} = \left[\begin{array}{ccc} \frac{\partial e_{f,t}}{\partial Z_{1,t}}, & ...., & \frac{\partial e_{f,t}}{\partial Z_{A,t}} \end{array} \right]$.

It is worth distinguishing between the effort of the representative vessel, $e_{f,t}$, and the effort of the fleet. Since each vessel operates during the maximum of T days and given the assumption on the measure of the vessels we have that $E_{f,t} = Te_{f,t}$, where we denote by $E_{f,t}$ the effort of the fleet. Taking this into account the yield by age of fleet f is given by

$$Y_{a,f,t} = \frac{p_{a,f}E_{f,t}}{Z_{a,t}}\left(1 - e^{-Z_{a,t}}\right)e^{x_{a,t}} = T\frac{p_{a,f}e_{f,t}}{Z_{a,t}}\left(1 - e^{-Z_{a,t}}\right)e^{x_{a,t}} \tag{8}$$

2.3 Equilibrium

The equilibrium requires that aggregate and disaggregate variables are consistent each other. Thus aggregated yield of the whole fishery must be equal to the sum of the yields by ages fishing by all fleets. That is, $Y_t = \sum_{f=1}^{N}\sum_{a=1}^{A} Y_{a,f,t}$.Taking into account the definition (8) and the optimal behavior condition of the vessels, (6), this equality implies that

$$Y_t = T\sum_{f=1}^{N} e_{f,t}\sum_{a=1}^{A}\frac{p_{a,f}}{Z_{a,t}}\left(1 - e^{-Z_{a,t}}\right)e^{x_{a,t}} = T\sum_{f=1}^{N} c_f e_{f,t}^2.$$

In the same way, aggregated yield by age of the whole fishery must be equal to the sum of the yields by ages of all fleets: $Y_{a,t} = \sum_{f=1}^{N} Y_{a,f,t}$.Taking into account the definitions (5) and (8), this equality implies that

$$p_a F_t = T\sum_{f=1}^{N} p_{a,f}e_{f,t}.$$

Substituting this into the mortality rate by age (2) we have

$$Z_{a,t} = F_{a,t} + m_a = p_a F_t + m_a = T\sum_{f=1}^{N} p_{a,f}e_{f,t} + m_a. \tag{9}$$

Using this relationship between mortality rate and fleets' efforts, we can solve the dynamics of the the fishery. Starting by linearizing (9) around the steady state, we can express the mortality rates by age in matrix form as a function of the technologies, P, and the policy variable, T. For the case of A ages and N fleets the expression is

$$\begin{bmatrix} \Delta Z_{1,t} \\ \Delta Z_{2,t} \\ \dots \\ \Delta Z_{A,t} \end{bmatrix} = T \begin{bmatrix} p_{1,1} & p_{1,2} & \dots & p_{1,N} \\ p_{2,1} & p_{2,2} & \dots & p_{2,N} \\ \dots & \dots & \dots & \dots \\ p_{A,1} & p_{A,2} & \dots & p_{A,N} \end{bmatrix} \begin{bmatrix} \Delta e_{1,t} \\ \Delta e_{2,t} \\ \dots \\ \Delta e_{N,t} \end{bmatrix}.$$

More compactly, with the obvious definitions

$$\Delta \mathbf{Z}_t = T\ \mathbf{P} \Delta \mathbf{e}_t.$$

Substituting here the optimal decisions of the fleets, equation (7), we can write

$$\Delta \mathbf{Z}_t = T\ \mathbf{P} \left[\mathbf{C}_t \Delta \mathbf{x}_t + \mathbf{D}_t \Delta \mathbf{Z}_t \right],$$

where $\mathbf{C}_t = \begin{bmatrix} C_{1,t} \\ C_{2,t} \\ \dots \\ C_{N,t}, \end{bmatrix}_{(N,A)}$ and $\mathbf{D}_t = \begin{bmatrix} D_{1,t} \\ D_{2,t} \\ \dots \\ D_{N,t}, \end{bmatrix}_{(N,A)}$.

Therefore, aggregate mortalities by age are given by

$$\Delta \mathbf{Z}_t = \Theta(T) \Delta \mathbf{x}_t, \tag{10}$$

where $\Theta_t(T) = \left(\frac{1}{T} \mathbf{I}_A - \mathbf{P} \mathbf{D}_t \right)^{-1} \mathbf{P} \mathbf{C}_t$ is a $(A \times A)$ matrix.

Summarizing, equations (4) and (10) represent the dynamics of the fishery after considering optimal behavior of the fleets and population dynamics. Substituting the aggregate age mortalities, equation (4), into the dynamics of the fishery, equation (10), the dynamics system can be represented by the following system

$$\Delta \mathbf{x}_{t+1} = \left[\mathbf{A} - \mathbf{B} \Theta_t(T) \right] \Delta \mathbf{x}_t. \tag{11}$$

Therefore, the dynamic system of the fishery can be understood as an homogeneous first order difference equation system. In the Appendix, the transition matrix of the dynamic system (11), $\left[\mathbf{A} - \mathbf{B} \Theta_t(T) \right]$, is shown more in detail.

3 Calibration

The multi-fleet fishery model is calibrated for the European Southern Stock of Hake for four species: hake (HKE), megrim whiffiagonis (MEG), megrim boscii (LDB), monkfish (MON).

We follow a two step procedure. First we run a vector autoregressive model of order 1, VAR(1), on recruitment data for the four species. A VAR model is used to capture the linear inter-dependencies among multiple time series. It is a generalization of the univariate autoregressive model. Each variable (recruitment) is represented by an equation explaining its evolution based on its own lags and the lags of the other variables (other species' recruitment). An estimated VAR model can be used for estimation, forecasting and to analyze causality between variables. The VAR(1) model estimated is

$$\mathbf{\Delta \tilde{x}}_t = \mathbf{\Psi} \mathbf{\Delta \tilde{x}}_{t-1} + \overrightarrow{\mathbf{e}}_t,$$

where $\mathbf{\Delta \tilde{x}}_t = \begin{pmatrix} HKE_t, & MEG_t, & LDB_t, & MON_t \end{pmatrix}'$ is the vector of the recruitment variables, $\overrightarrow{\mathbf{e}}_t = \begin{pmatrix} e_{1t}, & e_{2t}, & e_{3t}, & e_{4t} \end{pmatrix}'$ represents the vector of the structural shocks and $\mathbf{\Psi}$ is the (4×4) estimated coefficient matrix. The results of our estimation are

$$\begin{aligned} \widehat{\mathbf{\Psi}} &= \begin{pmatrix} 0.41098 & -0.0034348 & -0.22964 & 0.08625 \\ -0.014385 & 0.40748 & 0.98565 & 0.045343 \\ 0.059685 & -0.13282 & -0.12404 & 0.2796 \\ -0.76042 & -0.28971 & 0.31689 & 0.46978 \end{pmatrix}, \\ \Omega &= E(\overrightarrow{\mathbf{e}}_t, \overrightarrow{\mathbf{e}}'_t) = \begin{pmatrix} 0.0796 & -0.0663 & -0.0070 & -0.0259 \\ -0.0663 & 0.1960 & 0.0093 & -0.0094 \\ -0.0070 & 0.0093 & 0.0788 & 0.0488 \\ -0.0259 & -0.0094 & 0.0488 & 0.3956 \end{pmatrix}. \end{aligned}$$

Second, we estimate the feedback between the aggregate mortalities and the state variables, $\Theta(T)$. Note that the number of ages is 64. The R-squared = 0.9319 and the Adj R-squared = 0.8671.

4 Results

The state-space multi-fleet model was calibrated to match 2012 Fbar levels of the Southern Stock of Hake for four species: HKE, MEG, LDB and MON. Two scenarios were simulated. In the first scenario, fleets behavior was projected assuming that the response of the fleets will not be affected by management regulations. In the second scenario, the size of fluctuations is reduced to minimize (simultaneously) the distance of the four Fbars to the target.

Figure 1 shows the endogenous Fbar fluctuations generated by the model. The y-axis plots the deviation of Fbar from the target, and the x-axis, time. Note that differences in the recruitment variability (measured by the variance covariance matrix of the VAR process) and the existence of technical fishing interactions (captured in the state-space model) generate differences in the natural fluctuations of each species. Figure 2 summarizes all the simulations.

Figure 1: Endogenous Fbar fluctuations around the target. The y-axis and x-axis plot the deviation of Fbar from the target and the time, respectively. The horizontal lines represent the Fbar Ranges of each species.

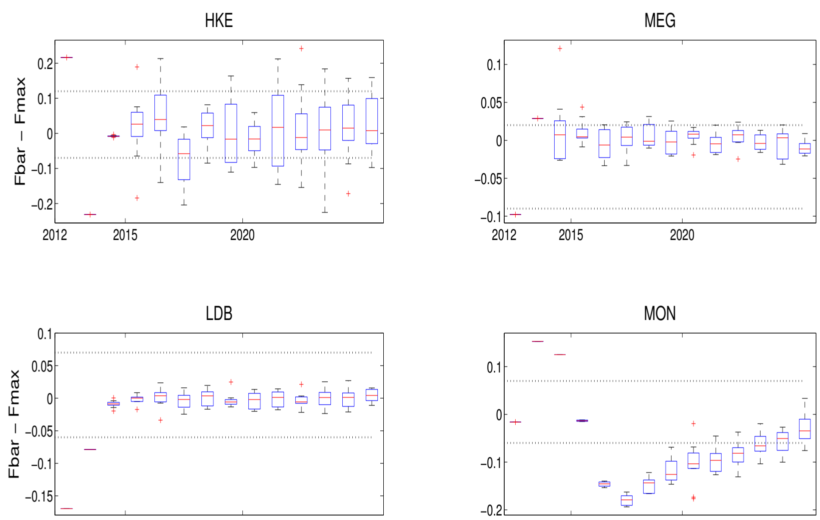

Figure 2: Endogenous Fbar. Time series 2012-2025. On each box, the central mark is the median; the edges of the box are the 25th and 75th percentiles. Black line is the mean. Horizontal blue lines represent Fbar Ranges

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Table 1: Fleet behavior I

age	Coef.	Std. Err.	t	$P>\|t\|$	[95% Conf.	Interval]
0						0 (omitted)
1	.9485	.5406837	1.75	0.084	-.1319701	202.897
2	2.126.452	.3602142	5.90	0.000	1.406.621	2.846.282
3	.0729414	.0104451	6.98	0.000	.0520686	.0938141
4	.0533637	.0074501	7.16	0.000	.0384758	.0682516
5	.0418973	.0058117	7.21	0.000	.0302835	.0535111
6	.0343405	.0047531	7.22	0.000	.0248422	.0438388
7	.029151	.0040307	7.23	0.000	.0210963	.0372057
8	.0252594	.0034906	7.24	0.000	.0182839	.0322348
9	.0222576	.0030749	7.24	0.000	.0161128	.0284024
10	.0198522	.002742	7.24	0.000	.0143727	.0253318
11	.0178923	.0024711	7.24	0.000	.0129543	.0228304
12	.0163074	.002252	7.24	0.000	.0118071	.0208076
13	.0149749	.0020679	7.24	0.000	.0108426	.0191072
14	.0138517	.0019127	7.24	0.000	.0100294	.0176739
15	.0129021	.0017815	7.24	0.000	.009342	.0164622
16	.4012811	.1928086	2.08	0.041	.0159839	.7865782
17	-.3377379	.5172804	-0.65	0.516	-137.144	.6959644
18	-.2905724	.3346845	-0.87	0.389	-.9593861	.3782413
19	-1.957.758	2.063.185	-0.95	0.346	-6.080.704	2.165.189
20	2.659.392	4.063.009	0.65	0.515	-5.459.883	1.077.867
21	.3843968	.221444	1.74	0.087	-.0581237	.8269173
22	.215054	.128183	1.68	0.098	-.0410993	.4712073
23						0 (omitted)
24	0	240.835	0.00	1.000	-4.812.704	4.812.704
25	.1777691	.2186185	0.81	0.419	-.2591051	.6146433
26	.1769586	.2166733	0.82	0.417	-.2560285	.6099456
27	1.504.663	.7248475	2.08	0.042	.0561706	2.953.155
28	.4361455	.2227782	1.96	0.055	-.0090413	.8813322
29	.1822508	.1000569	1.82	0.073	-.0176969	.3821985
30	.1005555	.0623745	1.61	0.112	-.02409	.2252009
31	.0652989	.0391849	1.67	0.101	-.0130059	.1436037

Table 2: Fleet behavior II

age	Coef.	Std. Err.	t	$P > \|t\|$	[95% Conf.	Interval]
32						0 (omitted)
33	-.280784	1.106.722	-0.25	0.801	-2.492.392	1.930.824
34	-.7345394	1.063.321	-0.69	0.492	-2.859.416	1.390.337
35	-.7063059	.393369	-1.80	0.077	-1.492.391	.0797793
36	-.714998	.4142859	-1.73	0.089	-1.542.882	.1128862
37	.0238363	.0117914	2.02	0.047	.0002731	.0473995
38	.0170322	.0084292	2.02	0.048	.0001878	.0338767
39	.0135896	.0066651	2.04	0.046	.0002705	.0269088
40	.0116152	.0057278	2.03	0.047	.000169	.0230613
41	.0103486	.0050193	2.06	0.043	.0003184	.0203789
42	.009027	.0044109	2.05	0.045	.0002124	.0178416
43	.0083894	.0041075	2.04	0.045	.0001812	.0165975
44	.008015	.0039031	2.05	0.044	.0002153	.0158147
45	.0076754	.0036982	2.08	0.042	.0002852	.0150657
46	.0080929	.0035259	2.30	0.025	.001047	.0151388
47	.0075415	.0033418	2.26	0.028	.0008635	.0142195
48	.0069665	.0031387	2.22	0.030	.0006943	.0132386
49	.0065067	.0029823	2.18	0.033	.0005471	.0124663
50	.006219	.0028959	2.15	0.036	.000432	.012006
51	.005987	.0028244	2.12	0.038	.0003429	.0116312
52	.0057523	.0027481	2.09	0.040	.0002605	.011244
53	.0055139	.002662	2.07	0.042	.0001944	.0108335
54	.005285	.0025804	2.05	0.045	.0001285	.0104416
55	.0051316	.002531	2.03	0.047	.0000737	.0101895
56	.0049787	.0024728	2.01	0.048	.0000372	.0099202
57	.0048031	.002405	2.00	0.050	-2.96e-06	.0096092
58	.004639	.0023369	1.99	0.051	-.0000308	.0093089
59	.0044511	.0022568	1.97	0.053	-.0000588	.0089609
60	.0042865	.0021854	1.96	0.054	-.0000806	.0086537
61	.0041348	.0021183	1.95	0.055	-.0000983	.008368
62	.0039943	.0020553	1.94	0.056	-.0001129	.0081015